

Transformed variable approach to filter approximation
(for Cameron's 1999 example)

dB := 1

Filter degree: $n := 4$

Number of finite transmission zeros: $n_{ftz} := 2$

Normalised frequencies of transmission zeros: $\omega_{tz1} := 1.3217$ $\omega_{tz2} := 1.8082$

Transmission zeros at $\omega=\infty$ ($z=1$): $m_3 := 1$ $m_4 := 1$

Passband return loss: $RL_{min} := 22\text{dB}$

Transform all finite transmission zeroes from ω -domain to z-domain

$$i := 1 .. n_{ftz}$$

$$m_j := \sqrt{\frac{\omega z_i - 1}{\omega z_i + 1}}$$

$$m^T = (0.37224 \quad 0.53647 \quad 1 \quad 1)$$

Create E + zF in product form:

$$E_{zF}(z) := \prod_{i=1}^n (m_i + z)^2$$

Find coefficients of E + zF polynomial:

$$EzF := E_{zF}(z) \text{ coeffs, } z \rightarrow$$

$$\begin{pmatrix} 0.03987822052043533116 \\ 0.52244294016054040335 \\ 2.9161331069950443855 \\ 9.055086390252307845 \\ 17.112135576054359001 \\ 20.168017985120709692 \\ 14.494819365432546936 \\ 5.8174189534688277131 \\ 1 \end{pmatrix}$$

The even part of E+zF is E and the odd part is zF. On Im(z) axis (passband), the roots of E interlace with those of zF.
 Creating E and zF polynomial coefficients:

$$i := 1, 3 \dots 2n + 1 \quad E_i := EzF_i \quad E^T = (0.03988 \ 0 \ 2.91613 \ 0 \ 17.11214 \ 0 \ 14.49482 \ 0 \ 1)$$

$$i := 2, 4 \dots 2 \cdot n \quad zF_i := EzF_i \quad zF^T = (0 \ 0.52244 \ 0 \ 9.05509 \ 0 \ 20.16802 \ 0 \ 5.81742)$$

Define the polynomials E and zF as functions in ω yielding the characteristic function to obtain the filter response functions

$$z(\omega) := \sqrt{\frac{\omega - 1}{\omega + 1}} \quad Ez(\omega) := \sum_{i=1}^{2n+1} (E_i \cdot z(\omega)^{i-1}) \quad zFz(\omega) := \sum_{i=1}^{2n-1} (zF_{i+1} \cdot z(\omega)^i)$$

$$Ksq(\omega) := \frac{1}{\left(\frac{RL_{min}}{10} \frac{1}{10} - 1\right)} \cdot \frac{1}{\left[1 - \left(\frac{zFz(\omega)}{Ez(\omega)}\right)^2\right]} \quad S21dB(\omega) := -10 \cdot \log(1 + Ksq(\omega)) \quad S11dB(\omega) := 10 \cdot \log\left(1 - \frac{1}{1 + Ksq(\omega)}\right)$$

$\omega := -3, -2.99 \dots 4$

