Microwave Lowpass Filters with a Constricted Equi-Ripple Passband

Improved lowpass filter performance is achieved by using a nonstandard prototype network

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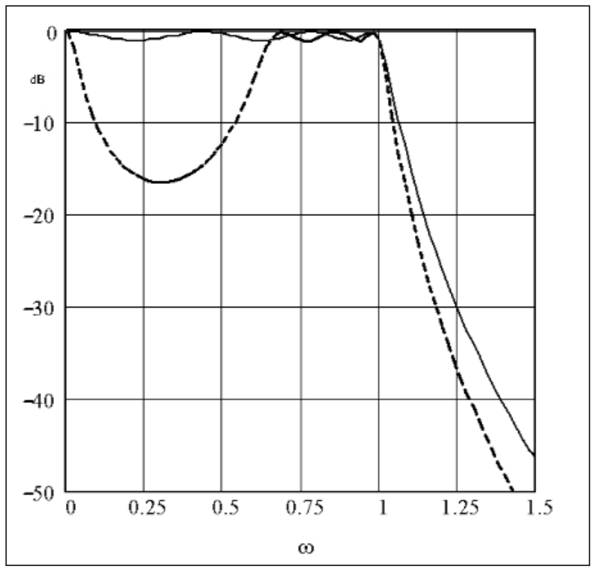
Lowpass filtering requirements in modern systems with RF bandwidth-limited signals, such as cellular, PCS and UMTS, usually demand lowpass-type selectivity. The standard DC-to-cutoff equi-ripple passband, however, is seldom required. Here, standard lowpass prototypes may be considered non-optimum or even wasteful in terms of an excessive passband width.

Lowpass functions with a finite-interval equi-ripple passband produce higher selectivity and allow control of the impedance level within the lowpass network [1]. Levy [1, 2, 9] has shown that special Zolotarev functions are suitable as quasi-lowpass approximation functions. Horton [3] has described Zolotarev quasi-elliptic lowpass filters with finite transmission zeros. However, Zolotarev functions are relatively complicated for run-of-the-mill lowpass filter design work, and easy-to-use filter tables have only been published for limited cases.

This paper presents a quasi-lowpass prototype network similar to a standard Chebyshev prototype, but with a constricted equi-ripple passband of variable width. A fast-converging iterative approximation method is introduced. Tables of normalized prototype elements (g-parameters) for various network degrees, passband ripples and passband widths are given for direct use in lowpass filter design at RF- and microwave frequencies. A detailed design example is given at the end.



▲ **Figure 1. Attenuation responses of the standard lowpass filter and the passband-constricted lowpass filter.**

Introduction

Lowpass filters are vital for modern communication systems with strict limits on unwanted signal radiation/transmission. Bandpass filters often require additional external lowpass filtering for suppression of spurious- and higher-order passbands. In these cases, the lowpass filter is only required to provide a true passband for the frequency range given by the passband of the preceding bandpass filter. Hence, it is not advantageous to start the lowpass filter design process with standard prototype networks with passbands from DC (zero) to a cutoff frequency ω_C. Moreover, the selectivity of a "constricted passband" lowpass prototype is superior to that of a standard lowpass prototype, and the physical size is smaller (Figures 1 and 3).

Approximation

The desired lowpass attenuation function associated with a realizable network is established by approximation. The reflective insertion loss of a lossless passive 2-port network is given by

$$\left|S_{21}(\omega)\right|^2 = \frac{1}{1+K^2(\omega)} \tag{1}$$

where $K(\omega)$ is referred to as either characteristic function, filtering function or reflection function. For standard lowpass approximations, such as the Chebyshev type, $K(\omega)$ is directly given as a hyperbolic-cosine function with inherent equi-ripple properties.

For a lowpass filter with all its transmission zeros at infinity, $K(\omega)$ is a polynomial in $j\omega$ and can be written as

$$K(j\omega) = \varepsilon(j\omega)^{\upsilon}\prod_{\mu=1}^{n-\upsilon}\left(\Omega_{\mu}{}^2+(j\omega)^2\right) \tag{2}$$

where υ = number of reflection zeros at DC (zero) and n = number of reflection zeros.

Here the reflection zeros Ω_μ can be placed freely between zero and the cutoff frequency ω_C to suit any desired passband reflection behavior. However, only a certain distribution of these reflection zeros will ensure an equi-ripple behavior within a finite frequency interval. Analytic equi-ripple-solutions involving special Zolotarev functions are available [1], and their application in commensurate microwave lowpass filters has been demonstrated [2].

A relatively simple and fast converging iteration process for generating a desired characteristic function directly can be found in bandpass filter approximation theory [6]. $K(j\omega)$ can be used for finding lumped lowpass filter elements, as well as for the synthesis of cascaded transmission line lowpass filters.

The iterative approximation process

Consider a characteristic function in the form of Equation (2). A linearized change ΔK of K [6] can be derived from the logarithmic form of Equation (2) as follows:

$$\Delta K(\omega) = \left[\frac{\partial K(\omega)}{\partial \varepsilon}\right]\Delta\varepsilon + \left[\frac{\partial K(\omega)}{\partial \Omega_1}\right]\Delta\Omega_1 + \ldots \\ \ldots + \left[\frac{\partial K(\omega)}{\partial \Omega_n}\right]\Delta\Omega_{n-\nu} \tag{3}$$

In the given case of a lowpass filter with $n - \nu$ reflection zeros in the passband, we obtain $n - \nu - 1$ reflection maxima plus two-band-edge frequencies giving a total of $n - \nu + 1$ critical frequencies where the reflection coefficient reaches exactly the passband ripple level. The unknown parameters in the characteristic function given by Equation (2) are $\varepsilon, \Omega_1 \ldots \Omega_{n-\nu}$ — a total of $n - \nu + 1$ parameters — which is a figure identical to the number of critical passband frequencies. Therefore, a system of $n - \nu + 1$ linear equations can be set up using Equation (3) with a set of starting values for the unknowns ε and Ω_μ. The starting values can be rough estimates (Equation (18) in [1] can be used for calculating starting values). Solving the equation system provides $\Delta\varepsilon$ and $\Delta\Omega_\mu$ for improving the starting values ε and Ω_μ with which a new iteration is then carried out. This process is repeated until the desired accuracy of $K(\omega)$ is achieved. Convergence is very fast; [a MathCAD version takes three seconds for six reflection zeros with an attenuation accuracy of 10^{-5} dB using 10 iterations]. The critical frequencies inside the passband (the reflection peak frequencies) depend on the reflection zeros. These peak frequencies are the zeros of the derivative of $K(\omega)$ within the passband and need to be calculated during each iteration cycle.

Synthesis

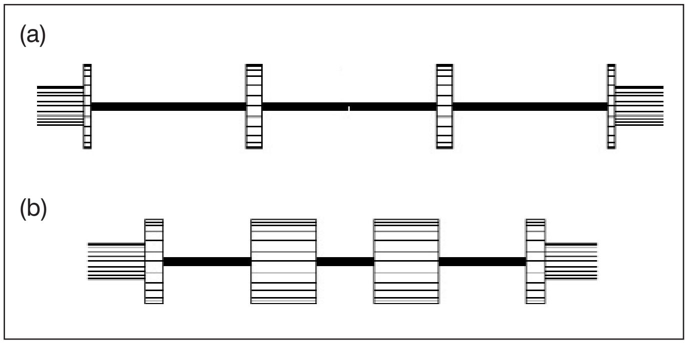
With the polynomial in Equation (2), the transfer function polynomial is given by the left s-plane roots of

$$H(s)H(-s) = 1 + K(s)K(-s) \tag{4}$$

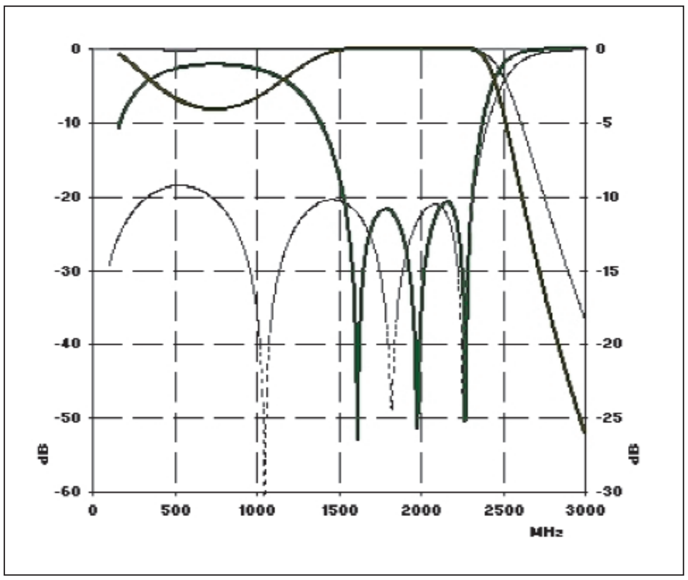
For both $K(s)$ being an odd- or even function, only an n-th degree polynomial needs to be rooted for finding the zeros of $H(s)$ [7]. With the two polynomials $K(s)$ and $H(s)$, the chain-matrix of the desired ladder network is given. From the chain-matrix elements the short- and open-circuit reactances are determined and normalized circuit elements can be extracted by continuous pole removal at $\omega = \infty$.

Tables of normalized elements

Tables of normalized element values are shown in Table 1 (see Appendix). These tables can be used to design lumped-element and distributed-element lowpass filters with constricted equi-ripple passbands. If n is odd, the characteristic function has a simple zero at DC (ν = 1 in (2)). For an even n, the characteristic function has a double zero at DC (ν = 2 in (2)); thus is avoided the problem of having non-zero attenuation at DC ($g_0 = g_{n+1} = 1$ for all cases). The double-zero also ensures that the network is by and large symmetric, which is not the case with even-degree Chebyshev prototypes. Separate tables are provided for two different passband ripple values and different lower equi-ripple, band-edge frequencies. The rightmost column contains stopband attenuation values at 1.5 times the cutoff frequency for each set of normalized element values. The process for finding denormalized elements is identical to that for the standard Chebyshev filters with the normalization frequency being the cutoff frequency ω_C.



▲ **Figure 2. (a) Conductor of the standard prototype design. (b) Conductor of the constricted equi-ripple passband lowpass filter design example.**



▲ **Figure 3. Lowpass filter response curves obtained from 3D EM simulation (left y-axis = *RL*, right y-axis = *IL*).**

Design example

A seventh degree coaxial lowpass filter was designed and simulated on the 3D EM simulator HFSS. The lower band edge was chosen to be at a normalized frequency of $\omega = 0.65$.

Example filter design specifications

Cutoff frequency:	2.3 GHz
Lower passband edge frequency:	1.495 GHz
Passband return loss:	20 dB
Port impedances:	50 Ω

A. Physical realization

Coaxial lowpass filters can be constructed in many different ways [4, 5]. A realization using quasi-lumped capacitors was chosen (Figure 2). The inductive sections are made of 150 ohm lines. A dielectric tube ($\varepsilon_r = 2.2$) supports the capacitive sections. Corrections for the fringing capacitances at the discontinuities are necessary [4, 8]. Here, the complication of having a "mixed dielectric" at the discontinuities was overcome by extracting exact fringing capacitance data from 3D EM-simulation *S*-parameter data.

Coaxial geometry data

Outer diameter:	6.0 mm
Inner diameter of dielectric tube:	5.6 mm
(this is also the OD of the quasi-lumped capacitors)	
Diameter of inductive sections:	0.5 mm

B. Electrical performance

Given the excellent agreement between 3D electromagnetic simulation and prototype measurement, the former was chosen to determine the electrical performance of the designed lowpass filter (thick traces in Figure 3).

C. Comparison with standard Chebyshev design

A standard seven-pole Chebyshev lowpass filter was designed for comparison with the constricted passband lowpass filter (CPL). In addition to better selectivity of the CPL filter (thin traces in Figure 3), there is also a clear size advantage. The length of the CPL design is only 75 percent of the length of the standard lowpass design. This means that higher degree CPL filters can be accommodated in the same space occupied by lower degree standard prototype lowpass filters.

Lowpass stopband requirements in popular communication systems often exist up to 13 GHz. The example CPL design provides a monotonic attenuation increase up to and above 13 GHz (109 dB at 13 GHz). In the standard prototype lowpass design, the attenuation slope changes its sign at 10 GHz and can only provide 63 dB attenuation at 13 GHz.

Conclusion

Lowpass filter prototypes with a constricted equi-ripple passband have distinct advantages over standard prototypes for most common microwave lowpass design applications. The required approximation can easily be accomplished by employing a flexible iterative method. A set of tables of normalized element parameters enables direct application of restricted equi-ripple passband lowpass prototypes in practical lowpass filter design work. ■

Acknowledgement

The author wishes to acknowledge that gathering of reflection zeros in lowpass filter functions was first suggested to him by B. v. Kameke in 1979.

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Author information

Dieter Pelz (Dipl. Ing) joined RFS Australia in 1995. He has been involved in microwave filter design for more than 20 years and holds four patents on filters and multiplexers. He may be reached via e-mail at dpelz@compuserve.com.

Appendix

n	g1	g2	g3	g4	g5	g6	g7	g8	g9	g10	Attenuation at =1.5
0.1 dB ripple / 16.4 dB RL				normalized lower equi-ripple passband edge at = 0.5							
3	1.03156	1.1474	1.03156								4.60 dB
4	non existent										
5	1.27439	1.28243	2.19193	1.28243	1.27439						20.73 dB
6	1.05042	1.51196	1.79566	1.79676	1.51146	1.05042					26.65 dB
7	1.57515	1.10489	3.15826	1.02307	3.15826	1.10489	1.57515				39.27 dB
8	1.22100	1.43926	2.04059	1.78699	1.78612	2.03917	1.43939	1.23588			45.00 dB
9	1.90936	0.88365	5.13993	0.54663	7.32761	0.54663	5.13993	0.88365	1.90936		57.90 dB
10	1.45985	1.25478	2.57245	1.47858	2.11793	2.11062	1.47955	2.56911	1.25455	1.47700	63.59 dB
0.044 dB ripple / 20 dB RL				normalized lower equi-ripple passband edge at = 0.5							
3	0.85495	1.10444	0.85495								2.61 dB
4	non existent										
5	1.06807	1.32100	1.94451	1.32100	1.06807						17.18 dB
6	0.90909	1.47702	1.72580	1.72648	1.47776	0.90731					23.07 dB
7	1.29823	1.21992	2.58993	1.21468	2.58993	1.21992	1.29823				35.67 dB
8	1.04493	1.45973	1.92370	1.78726	1.78847	1.92577	1.46026	1.03992			41.40 dB
9	1.56552	1.02581	3.83443	0.75501	5.03706	0.75501	3.83443	1.02581	1.56552		54.30 dB
10	1.22850	1.34127	2.27881	1.58697	2.03153	2.03653	1.58883	2.29055	1.34101	1.20923	60.00 dB
0.1 dB ripple / 16.4 dB RL				normalized lower equi-ripple passband edge at = 0.65							
3	1.08651	1.13655	1.08651								5.07 dB
4	0.92937	1.43463	1.43499	0.92838							9.59 dB
5	1.67051	1.02212	3.05011	1.02212	1.67051						23.65 dB
6	1.21803	1.43823	1.87832	1.87666	1.43834	1.22618					28.71 dB
7	2.48372	0.63281	8.47624	0.34639	8.47624	0.63281	2.48372				44.10 dB
8	1.73611	1.09373	2.82300	1.61314	1.60960	2.83516	1.09270	1.72573			49.12 dB
9	3.01774	0.47731	19.80094	0.10111	51.20188	0.10111	19.80094	0.47731	3.01774		64.58 dB
10	2.37530	0.72603	5.79318	0.76445	2.97182	3.02747	0.75557	5.78325	0.73139	2.33083	69.60 dB
0.044 dB ripple / 20 dB RL				normalized lower equi-ripple passband edge at = 0.65							
3	0.89562	1.10594	0.89562								2.94 dB
4	0.80064	1.35411	1.35360	0.80259							6.57 dB
5	1.34925	1.14466	2.46492	1.14466	1.34925						20.08 dB
6	1.03627	1.44707	1.79133	1.79032	1.44740	1.03067					25.12 dB
7	1.98884	0.77819	5.75880	0.51169	5.75880	0.77819	1.98884				40.51 dB
8	1.42857	1.21598	2.43672	1.66558	1.65881	2.44598	1.21681	1.41297			45.53 dB
9	2.50813	0.56856	13.76005	0.15077	32.16559	0.15077	13.76005	0.56856	2.50813		60.98 dB
10	1.96078	0.84981	4.56363	0.88918	2.71558	2.68475	0.89434	4.56979	0.84490	2.00569	66.00 dB

▲ **Table 1. Normalized prototype elements for lowpass filters with a constricted equi-ripple passband.**